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LETTER TO THE EDITOR

A note on the thermodynamic Bethe ansatz approach for excited states

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Abstract. We propose and analyse the thermodynamic Bethe ansatz equations for the first excited state in the minimal model M_5 perturbed by the subleading thermal field. Such state and the ground state have a different singular point in their scaling free energy.

Recently there has been renewed interest in the thermodynamic Bethe ansatz (TBA) approach in the context of deformed conformal field theories. The TBA method was first introduced by Yang and Yang [1] in the problem of the delta function interaction. This method takes advantage of the exact integrability by the Bethe ansatz, avoiding the difficult calculus of the partition function $Z = \mathrm{Tr}\,(e^{-H/T})$ at temperature T. Zamolodchikov [2] has pointed out that a relativistic version of TBA permits us to relate the free energy at temperature T to the Casimir energy [3–5] on a cylinder of circumference $R = 1/T$. Consequently, by studying the high temperature limit of the free energy, one is able to identify the central charge of the associated conformal field theory. Although there are also some results for the lowest excited states [6, 7], the complete characterization of the ultraviolet limit from the respective S-matrices is still an open problem. In particular, it is also interesting to investigate if the TBA method allows one to recover the conformal dimensions of different modular solutions [8–10]. In this letter we study this problem for the first example of universality classes in which the modular invariance matters, namely the minimal model M_5 ($c = \frac{4}{5}$) pertubed by the field $\phi_{2,1}$. We propose and analyse the TBA equations for the lowest odd state $(\frac{1}{40}, \frac{1}{40})$. Our motivation is based on the spectrum of the tetracritical Ising model (M_5) exhibited in [11]. Following this paper, one observes that the odd levels are enough to reproduce the massive spectrum both in the broken and unbroken symmetry phases.

The TBA equation that we propose is written as§

$$\varepsilon(\theta) + \frac{1}{2\pi}\int_{-\infty}^{\infty} d\theta' \psi(\theta - \theta') \log(1 + e^{-\varepsilon(\theta')}) = 2mR\cosh(\theta) \qquad (1)$$

where $\psi(x) = 4\sin(\frac{1}{3}\pi)\sinh(2x)/\sinh(3x)$, and m is the mass of the particle. The respective energy $E(R)$ is given by

$$E(R) = \frac{2\pi}{R} F \qquad F = \frac{-1}{4\pi^2} mR \int_{-\infty}^{\infty} d\theta \cosh(\theta) \log(1 + e^{-\varepsilon(\theta)}). \qquad (2)$$

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§ Recently, this equation has been also independently obtained by Fendly [7] in the context of twisted boundary conditions for the three-state Potts model. We stress that we are considering the minimal M_5 model with periodic boundary conditions.

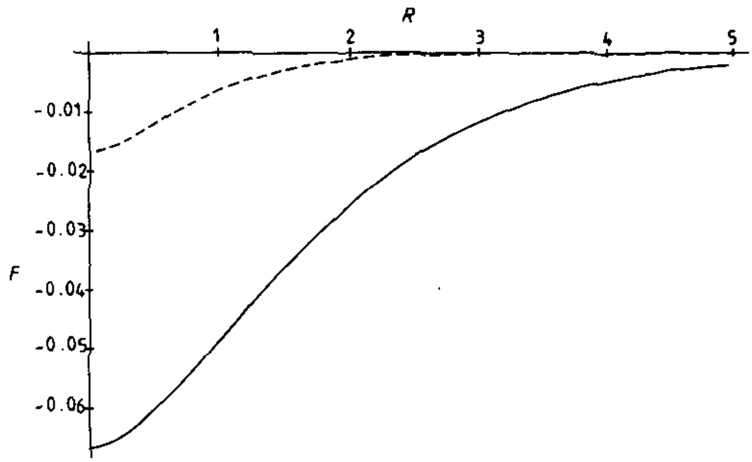

Figure 1. The ground state (full curve) and the first excited state (broken curve) in the M_5 model perturbed by the $\phi_{2,1}$ field.

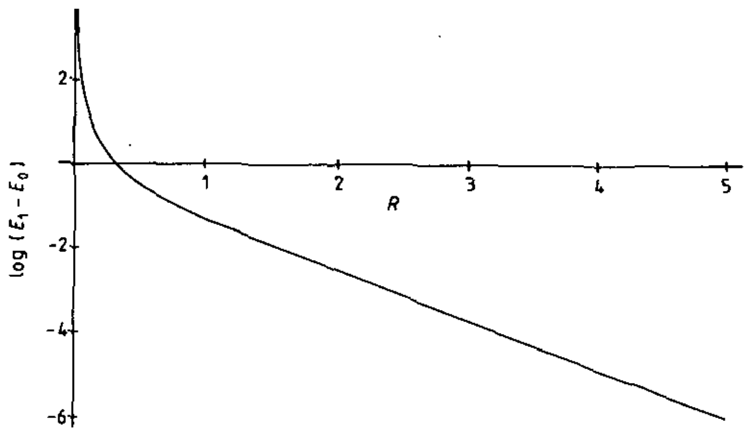

Figure 2. The exponential split $E_1(R)-E_0(R)\simeq e^{-mR}$.

Equation (1) is similar to that of the ground state in the three-state Potts model. The only change is the factor of two in the expression $mR\cosh(\theta)$, which we interpret as the degeneracy observed in the spectrum. The ultraviolet limit of (1) is straightforward, and the result is

$$E(R)=\frac{2\pi}{R}\left(\frac{2}{40}-\frac{4/5}{12}\right) \qquad R\to 0. \tag{3}$$

In order to analyse (1) we have solved it numerically for finite values of R. Figure 1 compares our solution $E_1(R)$ with the respective energy of the ground state $E_0(R)$. General arguments of quantum field theory [12] predict that, in the crossover region ($mR>1$), these two levels approach each other exponentially, $E_1(R)-E_0(R)\simeq e^{-mR}$. Figure 2 shows this exponential split, in which $m=1.01(\pm 1)$ in accordance with the normalization $m=1$.

Next step is to study the behaviour of (1) and (2) for small values of R. From general properties of (1) [13], one expects that the scaling function $F(mR)$ has a series

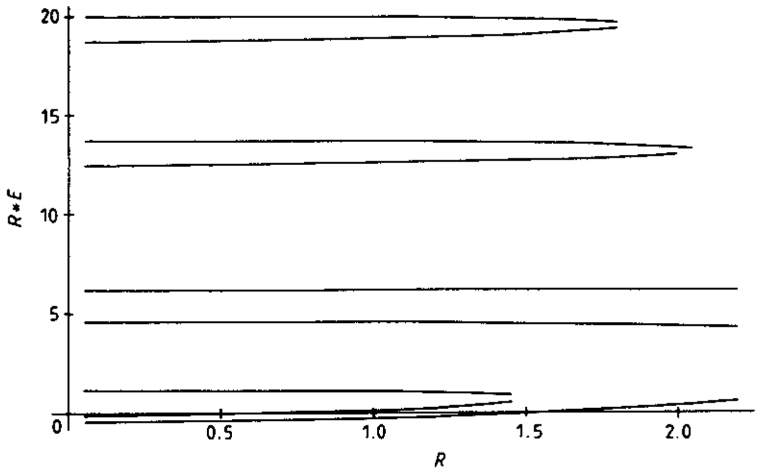

Figure 3. The spectrum of the M_5 model perturbed by the $\phi_{2,1}$ field for an imaginary coupling constant.

Table 1. The coefficients f_i for $i=1,2,3,4,5,6$ perturbing field.

i	f_i
1	−3.735 17E−02
2	1.892 55E−03
3	−3.498 1E−04
4	8.265E−05
5	−2.194E−05
6	6.13E−06

expansion in terms of the variable $g=r^{12/5}$ $(r=mR)$,

$$F(r)=-\tfrac{1}{60}+\varepsilon_0 r^2+\sum_{i=1}^{\infty} f_i g^i \tag{4}$$

where $\varepsilon_0=\sqrt{3}/6\pi$ is the bulk term.

We have fitted our numerical results with (4)†. This is in agreement with perturbation theory and the fusion rules for the field $(\frac{1}{40},\frac{1}{40})$. The first six coefficients f_i are collected in table 1. The convergence of (4) can be studied by analysing the singularity [2, 14] in the function F of the type

$$F\simeq(g-g_0)^\alpha \tag{5}$$

where the constants α and g_0 may be determined by the following formula

$$\frac{f_i}{f_{i-1}}=\frac{1}{g_0}\left(1-\frac{(\alpha+1)}{i}\right). \tag{6}$$

Using the coefficients of table 1, and extrapolating the final results, we find

$$g_0=-2.62(\pm1) \qquad \alpha=0.505(\pm2). \tag{7}$$

† We have also checked numerically for the general expansion $\Sigma_{i=1}^{\infty} f_i r^{6/5}$ that the first coefficients f_i, for odd i, are zero.

The value of g_0 can be compared with the similar singularity of the spectrum at imaginary coupling constant [15]. In figure 3 we plot such a spectrum using the truncated conformal approach [15–17]. We notice that the spectrum line coming from the field $(\frac{1}{40}, \frac{1}{40})$ becomes imaginary at $r_0 = g_0^{5/12} = 1.495(\pm 1)$, in accordance with the TBA results. The interesting point is that, although the ground state (identity operator) and the first excited state are degenerate, in the thermodynamic limit, they present a singularity at different points r_0 which may distinguish the odd and the even sectors.

Finally we present a few comments. It is very tempting a propose the generalization of (1) for other models with particle/antiparticle symmetry [7]. For instance, this will take into account the factor of two of (1) in the case of doublets (particle/antiparticle). However, we notice that if the spectrum also presents neutral particles, this straightforward generalization does not work. One typical example is the E_6 spectrum, constituted of two doublets and two singlets. We have checked that this trivial generalization of (1) and (2) does not even recover the exactly ultraviolet limit $(\frac{1}{56}, \frac{1}{56})$. Work in this direction is in progress.

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